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DIFFUSION OF A MEGAGAUSS FIELD INTO A METAL

S. F. Garanin, G. G. Ivanova,
D. V. Karmishin, and V. N. Sofronov

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The plane one-dimensional problem of the diffusion of a megagauss field into a metal wall is solved taking into account heat conduction and radiation transfer. At the interface, the magnetic field is assumed to be constant, and in this sense, the problem is close to the self-similar diffusion problem with parameters dependent on the self-similar variable $x/\sqrt{t}$. *It is shown that if heat conduction and radiation transfer are taken into account, in megagauss fields* (*in the examined formulation for fields* $B > 1.6$ MGs) *there is no loss of conductivity of the material evaporated by the magnetic field because of the formation of a plasma layer at the interface with a temperature in the electronvolt range. However, the role of the plasma layer in the structure of the skin layer remains insignificant up to fields* $B \approx 10$ MGs.

Key words: *diffusion, megagauss field, heat conduction.*

Introduction. The diffusion of megagauss fields into a metal plays an important role for both the generation of these fields and for their numerous applications, such as the acceleration of liners by a magnetic field, and has been studied beginning in the very early classical papers devoted to high magnetic fields. The theory of nonlinear magnetic diffusion into a metal is presented in a book by Knoepfel and other books and surveys [2, 3] and a broad range of related phenomena, such as metal vaporization and plasma formation was considered in the classical papers of Lyudaev [4]. Since then, problems of diffusion of high fields into a metal have been considered both experimentally and theoretically, in numerous papers and various applications, see, for example, [5]). However, in the literature there are no theoretical considerations of problems solved in the simplest formulation taking into account the main phenomena involved in the diffusion of a high magnetic field into a metal. Among these phenomena, plasma formation on a metal boundary is of significant importance.

The lack of clarity in the issues of diffusion of a high magnetic field into a metal leads to fallacies and inaccuracy in some papers. In many papers, including [4] there is the belief that explosion of a conductor leads to the formation of a cold nonconducting gas, which expands from the metal boundary across the field. However, from physical considerations and corresponding calculations it follows that in practice this does not occur in fields of about a few megagauss. Indeed, the radiation emitting from the surface of a hot metal with a temperature in the electronvolt range contains rigid quanta, which ionize the vapor formed and thus produce seed ionization. For low densities, the degree of this ionization near the vapor boundary should not depend on density. Thus, at the vapor boundary there is constant conductivity in the limit of arbitrarily low density. The formation of an electric field due to diffusion into the metal, which is increased by vapor motion across the magnetic field, causes Joule heating per unit volume that does not depend on the density and, hence, is infinitely large per unit mass for arbitrarily low density at the vapor–vacuum interface. This leads to an inevitable gas breakdown, formation of a plasma piston, which prevents gas expansion, and formation of a conducting plasma layer instead of the nonconducting expanding gas, as is confirmed by calculations.

Another fallacy that is often encountered in some papers is in a sense opposite to the first one. It is related to the belief that a rather hot and highly radiating plasma is formed at the boundary with the metal even in fields

Institute of Experimental Physics, Sarov 607190; sfgar@vniief.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 46, No. 2, pp. 5–12, March–April, 2005. Original article submitted June 25, 2004.

of about one megagauss — similarly to the magnetically pressed discharge on an insulator surface considered in [6]. This reasoning, however, does not apply to a plasma discharge on a metal surface since the shunting of the discharge over the plasma by metallic conductivity (even decreased because of Joule heating) sharply reduces the electric fields in the plasma, and, as a result, only a small fraction of the current flows over it even at fields of 10 MGs. For perfect metallic conductivity, discharge over the plasma is generally impossible since energy can be supplied to this discharge only from the magnetic field but the magnetic-field energy cannot decrease since the magnetic flux has no place to expand.

In many papers dealing with the motion of liners under the action of high magnetic fields, the corresponding magnetohydrodynamic (MHD) problems are considered ignoring heat conduction in plasma layers. In such consideration, numerical calculations can generally (for not too fine grids) give correct characteristics of liners but one should bear in mind that this approach contains internal inconsistencies and will not give correct results in the case of rather fine grids. Let us show this using a Lagrangian grid in a one-dimensional calculation.

In Lagrangian calculations ignoring heat conduction, the characteristic mass scale of the produced plasma ρx (ρ is the plasma density and x is the layer thickness) is determined by the grid resolution:

$$\rho x \sim \Delta m. \tag{1}$$

Therefore, we consider the plasma behavior on this scale if the characteristic magnetic B and electric E fields are determined by the diffusion into the metal adjacent to the plasma. The characteristic plasma pressure is determined by its Joule heating:

$$p \sim \sigma E^2 t, \tag{2}$$

where σ is the plasma conductivity and t is the characteristic time. For a thin plasma layer, the equilibrium condition should be satisfied with good accuracy:

$$p \sim (\sigma E/c) B x. \tag{3}$$

From (1) and (2) it follows that the layer thickness increases with time as

$$x \sim cEt/B. \tag{4}$$

Substituting the pressure $p \sim z\rho T$ (z is the degree of ionization of a multiply ionized plasma and T is the characteristic temperature) and the conductivity $\sigma \sim T^{3/2}/z$ into (2) and taking into account (1) and (4), we obtain

$$\frac{\sqrt{T}}{z^2} \sim \frac{B\Delta m}{E^3 t^2}. \tag{5}$$

For a multiply ionized plasma with $z \ll Z$ (Z is the nucleus charge), $z^{4/3} \sim T$ and from (5) we find that the plasma temperature $T \sim E^3 t^2/(B\Delta m)$ is inversely proportional to the grid resolution and increases with time until the plasma pressure $p \sim E^{17/4} t^{5/2}/(B(\Delta m)^{3/4})$ becomes equal to the magnetic pressure and the plasma shields the metal. If the plasma is heated to the level $z \sim Z$ and z is no longer dependent on temperature, the temperature rise becomes so rapid that it should be described by the differential form (2), i.e., $\rho\, dT/dt \sim T^{3/2} E^2$, and, using the equilibrium condition (3), for the temperature rise, we obtain

$$\frac{dT}{dt} \sim \frac{T^{5/4} E^{3/2}}{\sqrt{B\Delta m}}. \tag{6}$$

From (6) it follows that if the degree of ionization reaches the level $z \sim Z$, then, in the finite time $\tau \sim \sqrt{B\Delta m}/E^{3/2} T_0^{1/4}$ (T_0 is the temperature corresponding to the degree of ionization $z \sim Z$), the temperature goes to infinity, and the finer the grid the smaller this time. In fact, of course, the temperature increases until the thermal pressure of the plasma becomes equal to the magnetic pressure and until shielding of the metal occurs.

Thus, using rather fine grids in calculations, it is possible to obtain plasma shielding of the skin layer in the metal. In many cases for real grids, this shielding may not have time to develop in times of interest. Since for fields $B < 10$ MGs and correct accounting for the plasma region, the role of shielding in the current branched from the metal and the plasma mass confined in the skin layer is insignificant, the calculation error (even by several times) may have an insignificant effect on the liner behavior. In any case, however, one needs to know how to estimate the characteristics of the plasma layers and to understand that their incorrect account can lead to wrong results.

Formulation of the One-Dimensional Problem. The diffusion of a magnetic field into a metal was considered using as an example diffusion from vacuum into a semi-infinite copper wall. The calculations were performed in a one-dimensional MHD formulation on a Lagrangian grid. It was assumed that all values depend on the coordinates x and time t, and the magnetic B and electric E fields are perpendicular to each other and to the x axis. It was assumed that at the initial time, cold copper occupies the region $x > 0$, the magnetic field in this region is equal to zero, and a magnetic field as a function of time $B_0(t)$ was specified at the boundary of the material. The calculations took into account hydrodynamic motion, magnetic diffusion, electronic heat conduction, and radiative heat transfer in the "back and forth" approximation. The equation of state, conductivity, electronic heat conductivity, and radiation paths for copper used in the calculations are given in [7].

As regards the boundary condition that defines radiation propagation, two versions are possible: in one case, it is assumed that the entire radiation leaves the surface (an open system), and in the other cases, the radiation flux on the boundary is equal to zero (a closed system), which is possible if a magnetic field diffuses from a cavity whose walls are under identical conditions. Most of the calculations were performed for the open-system formulation and only some (for comparison) for the closed-system formulation.

In most problems, the case of a constant magnetic field on the boundary with a plasma $B_0 = \text{const}$ was considered. The problem thus becomes nearly self-similar, and, therefore, the profiles of all quantities are easily recalculated from one time to other times. Indeed, for real, not too small times (in excess of a few millimicroseconds), the hydrodynamic motion is far faster than the diffusion and it can be assumed that the total (thermal plus magnetic) pressure has time to level off over the region of the skin layer. In this case, the magnetic diffusion and heat conduction should be such that all values depend only on the self-similar variable $x/\sqrt{t}$. In principle, a deviation from this self-similar dependence could be produced by radiation transfer in the phase where the radiation path becomes comparable to the thickness of the plasma layer. In practice, calculations for $B_0 = \text{const}$ provide a good fit to the self-similar dependence.

The calculations ignored some phenomena that could basically affect the pattern of magnetic-field diffusion. First, the employed equation of state did not incorporate two-phase (liquid–vapor) states. Decay into phases occurred automatically in the calculations but only if the material fell in the thermodynamically unstable region $(\partial p/\partial \rho)_T < 0$, and, therefore, the states of an overheated liquid and supercooled vapor were allowed. As a result, the calculations ignored metal vaporization to vacuum for rather low-level fields $B_0 < 1.5$ MGs, for which there may be no plasma formation. However, the contribution of this effect is minor. Calculations with two-phase equations of state show that for fields $B_0 \approx 1$ MGs, not more than a few percent of the skin layer are evaporated.

Second, radiation transfer was considered in a gray material approximation, which could not provide a detailed account of the gas-breakdown and plasma-formation phenomena dealt with in the introduction. These subtle phenomena can be of special interest in studies of plasma formation at rather low-level fields $B_0 < 1.5$ MGs. However, as was mentioned above, these effects are inherent to a small fraction of mass and are generally not too important for the description of field diffusion into a metal.

Third, the dependence of the electrical conductivity and thermal conductivity on the degree of plasma magnetization and thermoelectric phenomena (Nernst effect) were ignored. Generally, these effects could influence the plasma behavior near the boundary with the vacuum, in the zone where the radiation transfer is not yet very important because in this region the degree of magnetization of electrons $\omega_e \tau_e$ can be about unity. However, this zone is a small fraction of the entire plasma layer, in which the role of radiation is mostly significant, and, hence, inaccuracy in the description of this zone hardly affects the description of the skin layer in the metal as a whole.

Results of Open-System Calculations for a Constant Magnetic Field on the Boundary. Calculated profiles of the magnetic field $B(x)$ and the density $\rho(x)$ and temperature $T(x)$ of the material for $B_0 = 1$, 2, 5, and 10 MGs at $t = 1$ μsec are presented in Fig. 1, from which one can see how the structure of the skin layer varies as the magnetic field increases. For $B_0 = 1$ MGs, the copper present in the skin layer is only in the condensed phase. For $B_0 = 2$ MGs, the skin layer contains not only the condensed phase but also a two-phase liquid–vapor region (on the plot given in Fig 1b, the density fluctuations in the two-phase region are smoothed) and a plasma region, which can also be divided into a zone of radiative heat conduction and a zone of electronic heat conduction at the boundary with the vacuum, in which the radiation is almost insignificant. Calculations in the formulation considered (an open system; $B_0 = \text{const}$) showed that transition from the single-phase structure of the skin layer (Fig. 1a) to a composite multiphase structure (Fig. 1b) occurs for approximately $B_0 = 1.6$ MGs. As the magnetic

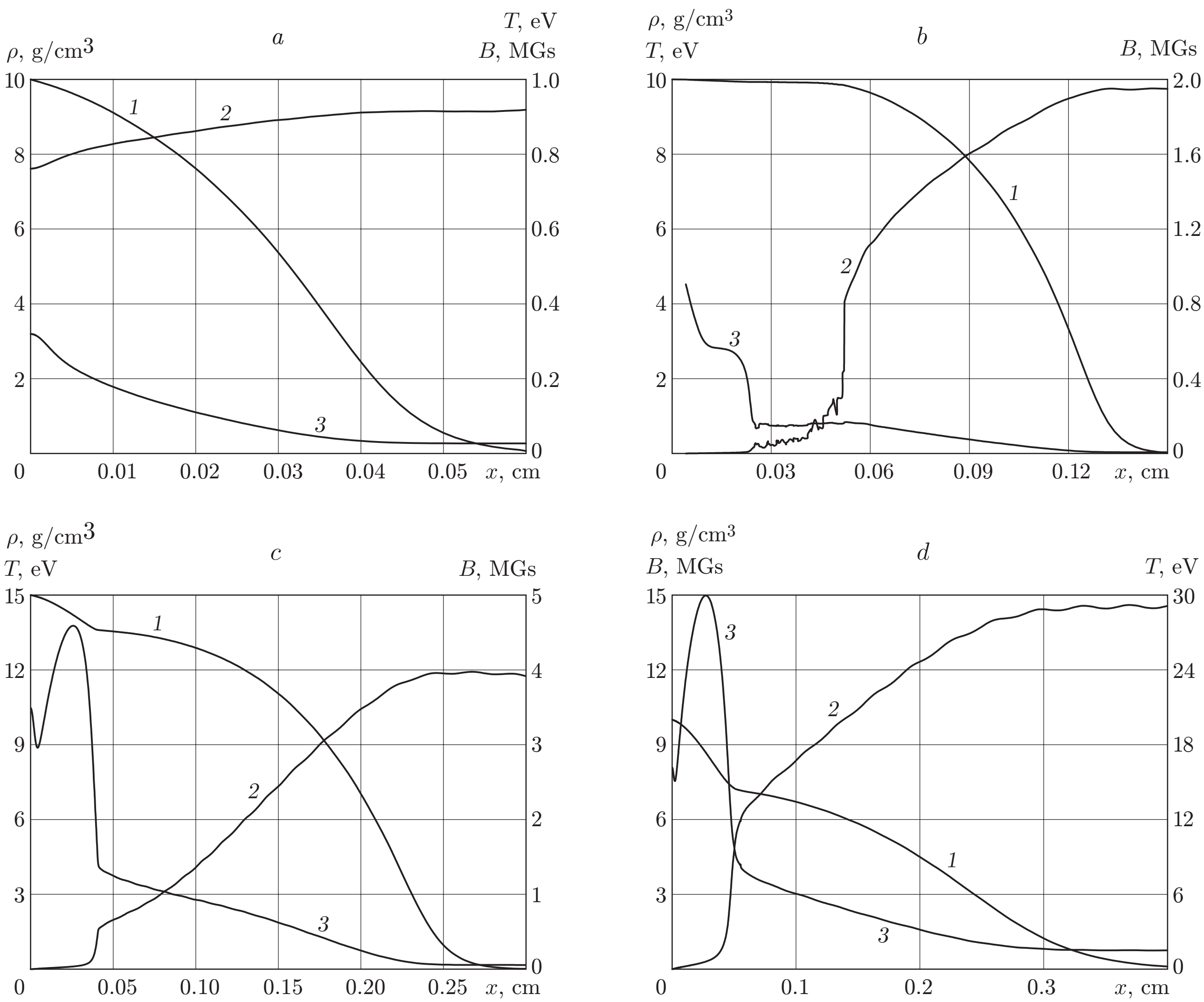

Fig. 1. Spatial curves of the magnetic field $B(x)$ (1), material density $\rho(x)$ (2), and material temperature $T(x)$ (3) calculated for an open system with a constant magnetic field on the boundary $B_0 = 1$ (a), 2 (b), 5 (c), and 10 MGs (d) at $t = 1$ μsec.

field B_0 increases, the two-phase region in the skin layer disappears and for large fields, the skin layer (Fig. 1c and d) consists only of a condensed phase and a plasma region, in which it is possible to distinguish a zone of radiative heat conduction (with a temperature decreasing toward the vacuum, which is explained by plasma cooling due to the radiation transmitted through the surface) and a zone of electronic heat conduction with a temperature increasing toward the vacuum. It should be noted that as shown in Fig. 1d, for $B_0 = 10$ MGs, a rather large contribution to the heating of the material (commensurable with the Joule heating) in the dense region comes from shock-wave heating, which is substantial for high fields in this formulation, in which the magnetic field is applied to the surface instantaneously.

For all fields $B_0 \leqslant 10$ MGs, the plasma region is insignificant and is a small fraction of the skin layer. This is also confirmed by the data of Table 1, which gives the skin-layer thickness $x(t) = \dfrac{1}{B_0}\displaystyle\int B\,dx$ and its mass $m(t) = \dfrac{1}{B_0}\displaystyle\int B\rho\,dx$ at $t = 1$ μsec for the fields considered. A comparison of the indicated values shows that for these fields, the mean material density in the skin layer is rather high and corresponds to the density of the condensed phase (although it is hardly possible to speak of a condensed phase in a material heated strongly by a shock-wave

TABLE 1

Magnetic field B_0, MGs	$x(t) = \frac{1}{B_0}\int B\,dx$, cm	$\frac{x(1\ \mu\text{sec})}{x(0.1\ \mu\text{sec})\sqrt{10}}$	$m(t) = \frac{1}{B_0}\int B\rho\,dx$, g/cm^2	$\frac{m(1\ \mu\text{sec})}{m(0.1\ \mu\text{sec})\sqrt{10}}$	Fraction of current branched in plasma region, %
1	0.0297	1.00	0.252	1.00	0
2	0.106	1.03	0.416	1.00	0.7
5	0.175	1.03	0.797	1.01	9
10	0.168	1.03	1.31	1.04	25

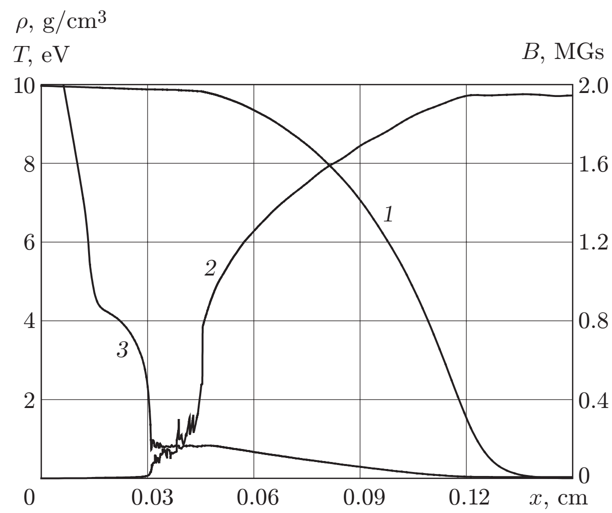

Fig. 2. Spatial curves of the magnetic field $B(x)$ (1), material density $\rho(x)$ (2), and material temperature $T(x)$ (3) calculated for a closed system with a constant magnetic field on the boundary $B_0 = 2$ MGs at $t = 1$ μsec.

with $B_0 = 10$ MGs). The skin-layer thickness as a function of B_0 increases rapidly from 1 to 2 MGs because of nonlinear diffusion and the appearance of the two-phase region and the plasma region. Then for large fields, the skin-layer thickness increases more slowly, and in the range 5–10 MGs, the increase ceases because magnetic-field amplification leads to an increase in the material density in the skin layer, including in the plasma region. It is interesting that with increase in the field in the range 1–10 MGs, the mass of the skin layer increases monotonically, approximately as $m \sim B_0^{0.72}$. According to the data of Table 1, the fraction of the current branched in the plasma region is insignificant for fields $B_0 \leqslant 5$ MGs and only for $B_0 = 10$ MGs, it has an appreciable value.

Let us consider how the real time dependence of the quantities in the skin is close to the self-similar one, in which all quantities should depend only on the ratio $x/\sqrt{t}$. The values of $x(t_2)\sqrt{t_1}/x(t_1)\sqrt{t_2}$ [$x(t)$ is the skin-layer, and t_1 and t_2 are different times] and $m(t_2)\sqrt{t_1}/m(t_1)\sqrt{t_2}$ [$m(t)$ is the mass of the skin layer] given in Table 1 can serve as a measure of deviation from this dependence; for rigorous self-similarity, they should be equal to unity. The data of Table 1 show that, indeed, for all fields considered, the dynamics of the skin layer in this formulation is nearly self-similar. Small deviations from self-similarity are explained by the greater role of radiation with increase in time, resulting in a decrease in the temperature of the plasma region and, hence, an increase in its relative thickness.

Effect of the Boundary Conditions for the Radiation on the Structure of the Skin Layer. We consider how the structure of the skin layer changes if to the radiation flux on the boundary is set equal to zero (a closed system). This situation is exemplified by magnetic-flux compression in a cavity. Calculated profiles of the magnetic field $B(x)$, material density $\rho(x)$, and material temperature $T(x)$ corresponding to this case for $t = 1$ μsec and $B_0 = 2$ MGs are presented in Fig 2.

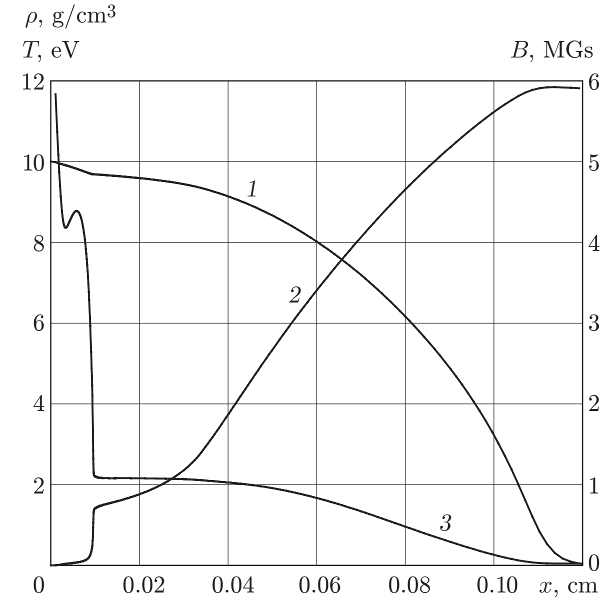

Fig. 3. Spatial curves of the magnetic field $B(x)$ (1), material density $\rho(x)$ (2), and material temperature $T(x)$ (3) calculated for an open system with a linearly increasing magnetic field at the boundary for $dB_0/dt = 5$ MGs/μsec at $t = 1$ μsec.

As one might expect, a comparison of Fig. 2 and Fig. 1b shows that the temperature in the plasma region is slightly higher in closed-system calculations (in the zone of radiative heat conduction in the case presented in Fig. 1b, it is about 3 eV, and in Fig 2, it is about 4 eV). In addition, there is a decrease in the thickness of the two-phase zone in the closed system. As a result, the thickness of the skin layer is slightly higher in closed-system calculations than in open-system calculations. However, the masses of the skin layer in both cases are approximately identical (in the closed system, it is 0.6% lower).

Effect of a Smooth Increase in the Magnetic Field on the Structure of the Skin Layer. The calculations described here assumed that the magnetic field is instantaneously applied to the metal boundary and then remains constant. In most real problems, the magnetic field at the boundary increases gradually, and this, of course, changes the structure of the skin layer. In the case of smoothly increasing field for moderately high fields, the magnetic diffusion into metal can be calculated ignoring heat conduction, which cannot be done for instantaneous switching of the field. Indeed, for instantaneous switching of the field for the self-similar law of variation in the electric field at the boundary, we have $E \sim 1/\sqrt{t}$ and the integral over time corresponding to the Joule heating at the boundary diverges for small times. Therefore, the heating of the material near the boundary needs to be described taking into account heat conduction, which distributes the heat released near the boundary over a certain region. As a result, in problems of diffusion of moderately high field (up to 1 MGs) into a metal, the volumetric Joule heating for a smoothly increasing field is equal to approximately $B^2/(8\pi)$ [1], whereas for instantaneous switching of the field near the boundary, it is much larger (for the case given in Fig 1a, by a factor of approximately 2.6).

To illustrate how a smooth increase in the magnetic field at the boundary affects the structures of the skin layer in megagauss fields, Fig. 3 gives profiles of the magnetic field $B(x)$, material density $\rho(x)$, and material temperature $T(x)$ at $t = 1$ μsec calculated for a magnetic field increasing linearly with time for $dB_0/dt = 5$ MGs/μsec, so that at $t = 1$ μsec the magnetic field on the boundary is equal 5 MGs. A comparison of Figs. 3 and Fig. 1c shows that in the case of megagauss fields, the heating of the skin layer is also smaller for a smoothly increasing magnetic field than for instantaneous switching. Accordingly, a plasma layer is formed for higher magnetic fields in the case of a smoothly increasing magnetic field than in the case of instantaneous switching. In the present calculation, plasma formation occurred when the magnetic field at the boundary reached a value of 3 MGs, which is almost twice larger than that for instantaneous switching.

Conclusions. Explosion of a conductor for fields in excess of $B \approx 1.5$–3 MGs leads to formation of a conducting plasma layer at the boundary with the vacuum. For fields $B < 10$ MGs, the role of this layer in the current branched from the metal and the plasma mass confined in the skin layer is insignificant but is of fundamental importance since incorrect account for it (for example, in numerical calculations without heat conduction on rather fine grids) can lead to complete branching of the current into the plasma layer. For a correct description of the skinning of megagauss fields in a metal, one needs to take into account electronic heat conduction and radiative heat transfer.

For magnetic fields at the metal boundary in excess of $B_0 \approx 1.5$–3 MGs, the skin layer consists of a condensed-phase region with a nearly initial density, a two-phase liquid–vapor region, and a plasma region, which can also be divided into a region of radiative heat conduction and a region of electronic heat conduction at the boundary with the vacuum. A two-phase liquid–vapor region is formed for fields $B_0 \approx 1.5$–4 MGs, depending on the dynamics of the magnetic field at the boundary and the boundary conditions for the radiation.

Numerical calculations of megagauss-field diffusion with a constant magnetic field at the boundary $B_0 = \text{const}$ showed that for all fields in the range $B < 10$ MGs for times larger than a few millimicroseconds, the dependence of all quantities in the skin layer is adequately described by a self-similar dependence (on the variable $x/\sqrt{t}$).

A comparison of closed- and open-system calculations shows that the temperature in the plasma region in a closed system is slightly higher (for example, for $B_0 = 2$ MGs in the zone of radiative heat conduction in the open system it was about 3 eV, and in the closed system, it was about 4 eV).

The heating of the skin layer is considerably higher for a smoothly increasing magnetic field than for instantaneous switching. Accordingly, in this case, the formation of a plasma layer occurs at higher magnetic fields than it does in the case of instantaneous switching.

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